Set 16: The pH scale

1. (a) $[H^+] = [HC\ell] = 0.100 \text{ mol L}^{-1}$
$k_W = [H^+]\,[OH^-] = 1.00 \times 10^{-14}$
$0.100 \times [OH^-] = 1.00 \times 10^{-14}$
$[OH^-] = 1.00 \times 10^{-13} \text{ mol L}^{-1}$
$pH = -\log_{10}[H^+] = -\log_{10}(0.100) = 1.00$

(b) $[H^+] = [HNO_3] = 0.00500 \text{ mol L}^{-1}$
$k_W = [H^+]\,[OH^-] = 1.00 \times 10^{-14}$
$0.00500 \times [OH^-] = 1.00 \times 10^{-14}$
$[OH^-] = 2.00 \times 10^{-12} \text{ mol L}^{-1}$
$pH = -\log_{10}[H^+] = -\log_{10}(0.00500) = 2.30$

(c) $[OH^-] = [NaOH] = 0.0100 \text{ mol L}^{-1}$
$k_W = [H^+]\,[OH^-] = 1.00 \times 10^{-14}$
$[H^+] \times 0.0100 = 1.00 \times 10^{-14}$
$[H^+] = 1.00 \times 10^{-12} \text{ mol L}^{-1}$
$pH = -\log_{10}[H^+] = -\log_{10}(1.00 \times 10^{-12}) = 12.0$

(d) $[H^+] = [HC\ell] = 2.00 \text{ mol L}^{-1}$
$k_W = [H^+]\,[OH^-] = 1.00 \times 10^{-14}$
$2.00 \times [OH^-] = 1.00 \times 10^{-14}$
$[OH^-] = 5.00 \times 10^{-15} \text{ mol L}^{-1}$
$pH = -\log_{10}[H^+] = -\log_{10}(2.00) = 0.300$

(e) Solutions containing Na^+ ions (derived from a strong base) and $C\ell^-$ ions (derived from a strong acid) do not hydrolyse in water. The solution is therefore neutral so
$[H^+] = [OH^-] = 1.00 \times 10^{-14} \text{ mol L}^{-1}$ and the pH = 7.00

2. (a) $pH = -\log_{10}[H^+]$
$3.00 = -\log_{10}[H^+]$
$[H^+] = 1.00 \times 10^{-3} \text{ mol L}^{-1}$
$k_W = [H^+]\,[OH^-] = 1.00 \times 10^{-14}$
$1.00 \times 10^{-3} \times [OH^-] = 1.00 \times 10^{-14}$
$[OH^-] = 1.00 \times 10^{-11} \text{ mol L}^{-1}$

(b) $pH = -\log_{10}[H^+]$
$11.0 = -\log_{10}[H^+]$
$[H^+] = 1.00 \times 10^{-11} \text{ mol L}^{-1}$
$k_W = [H^+]\,[OH^-] = 1.00 \times 10^{-14}$
$1.00 \times 10^{-11} \times [OH^-] = 1.00 \times 10^{-14}$
$[OH^-] = 1.00 \times 10^{-3} \text{ mol L}^{-1}$

2 (c) $pH = -\log_{10}[H^+]$
$-1.00 = -\log_{10}[H^+]$
$[H^+] = 10.0 \text{ mol L}^{-1}$
$k_W = [H^+]\,[OH^-] = 1.00 \times 10^{-14}$
$10.0 \times [OH^-] = 1.00 \times 10^{-14}$
$[OH^-] = 1.00 \times 10^{-15} \text{ mol L}^{-1}$

(d) $pH = -\log_{10}[H^+]$
$4.56 = -\log_{10}[H^+]$
$[H^+] = 2.75 \times 10^{-5} \text{ mol L}^{-1}$
$k_W = [H^+]\,[OH^-] = 1.00 \times 10^{-14}$
$2.75 \times 10^{-5} \times [OH^-] = 1.00 \times 10^{-14}$
$[OH^-] = 3.63 \times 10^{-10} \text{ mol L}^{-1}$

(e) $pH = -\log_{10}[H^+]$
$7.60 = -\log_{10}[H^+]$
$[H^+] = 2.51 \times 10^{-8} \text{ mol L}^{-1}$
$k_W = [H^+]\,[OH^-] = 1.00 \times 10^{-14}$
$2.51 \times 10^{-8} \times [OH^-] = 1.00 \times 10^{-14}$
$[OH^-] = 3.98 \times 10^{-7} \text{ mol L}^{-1}$

3. For the acid:
$pH = -\log_{10}[H^+]$
$4.00 = -\log_{10}[H^+]$
$[H^+] = 1.00 \times 10^{-4} \text{ mol L}^{-1}$

For the neutral solution:
$pH = -\log_{10}[H^+]$
$7.00 = -\log_{10}[H^+]$
$[H^+] = 1.00 \times 10^{-7} \text{ mol L}^{-1}$

Concentration changed by a factor of $\dfrac{1.00\times10^{-4}}{1.00\times10^{-7}} = 1.00\times10^{3} = 1000$

4. $pH = -\log_{10}[H^+]$
$2.00 = -\log_{10}[H^+]$
$[H^+] = 1.00 \times 10^{-2} \text{ mol L}^{-1}$

$n(H^+)_{\text{in 2 L of depleted soln}} = cV = 1.00 \times 10^{-2} \times 2.00 = 2.00 \times 10^{-2} \text{ mol}$
$n(H^+)_{\text{in 3 M soln}} = cV = 3.00 \times 3.00 = 9.00 \text{ mol}$
$n(H^+)_{\text{total in new soln}} = n(H^+)_{\text{in 2 L of depleted soln}} + n(H^+)_{\text{in 3 M soln}} = 2.00 \times 10^{-2} + 9.00 = 9.02 \text{ mol}$

$[H^+]_{\text{in new soln}} = \dfrac{n}{V} = \dfrac{9.02}{5} = 1.80 \text{ mol L}^{-1}$

5. $pH = -\log_{10}[H^+]$
$5.00 = -\log_{10}[H^+]$
$[H^+] = 1.00 \times 10^{-5}$ mol L^{-1}

$pH = -\log_{10}[H^+]$
$3.60 = -\log_{10}[H^+]$
$[H^+] = 2.51 \times 10^{-4}$ mol L^{-1}

$c_1V_1 = c_2V_2$
$2.51 \times 10^{-4} \times 25.0 \times 10^{-3} = 1.00 \times 10^{-5} \times V_2$
$V_2 = 0.628$ L = 628 mL

Water required = 628 – 25.0 = 603 mL

6. $pH = -\log_{10}[H^+]$
$12.0 = -\log_{10}[H^+]$
$[H^+] = 1.00 \times 10^{-12}$ mol L^{-1}
$k_W = [H^+]\,[OH^-] = 1.00 \times 10^{-14}$
$1.00 \times 10^{-12} \times [OH^-] = 1.00 \times 10^{-14}$
$[OH^-] = 1.00 \times 10^{-2}$ mol L^{-1}

$pH = -\log_{10}[H^+]$
$11.7 = -\log_{10}[H^+]$
$[H^+] = 1.995 \times 10^{-12}$ mol L^{-1}
$k_W = [H^+]\,[OH^-] = 1.00 \times 10^{-14}$
$1.995 \times 10^{-12} \times [OH^-] = 1.00 \times 10^{-14}$
$[OH^-] = 5.01 \times 10^{-3}$ mol L^{-1}

$n(OH^-)_{target\ soln} = cV = 1.00 \times 10^{-2} \times 0.100 = 1.00 \times 10^{-3}$ mol
$n(OH^-)_{pH11.7\ soln} = cV = 5.01 \times 10^{-3} \times 0.100 = 5.01 \times 10^{-4}$ mol
$n(OH^-)_{to\ be\ added} = n(OH^-)_{target\ soln} - n(OH^-)_{pH11.7\ soln} = 1.00 \times 10^{-3} - 5.01 \times 10^{-4} = 4.99 \times 10^{-4}$ mol

$n(NaOH)_{to\ be\ added} = n(OH^-)_{to\ be\ added} = 4.99 \times 10^{-4}$ mol

$M(NaOH) = 39.998$ g mol^{-1}
$m(NaOH)_{to\ be\ added} = nM = 4.99 \times 10^{-4} \times 39.998 = .00200$ g = 20.0 mg

7. (a) $n(OH^-)_{drain\ water} = cV = 0.236 \times 0.200 = 0.0472$ mol
$n(OH^-)_{runoff\ water} = cV = 0.156 \times 0.300 = 0.0468$ mol
$n(OH^-)_{total} = n(OH^-)_{drain\ water} + n(OH^-)_{runoff\ water} = 0.0472 + 0.0468 = 0.0940$ mol

$$[OH^-]_{mixed\ water} = \frac{n(OH^-)_{total}}{V_{total}} = \frac{0.0940}{0.500} = 0.188 \text{ mol L}^{-1}$$

$k_W = [H^+]\,[OH^-] = 1.00 \times 10^{-14}$
$[H^+] \times 0.188 = 1.00 \times 10^{-14}$
$[H^+] = 5.32 \times 10^{-14}$ mol L^{-1}
$pH = -\log_{10}[H^+] = -\log_{10}(5.32 \times 10^{-14}) = 13.3$

(b) $n(OH^-)_{in\ 1\ L} = cV = 0.188 \times 1.00 = 0.188$ mol

$H^+(aq) + OH^-(aq) \leftrightharpoons H_2O(\ell)$
$n(HC\ell) = n(H^+) = n(OH^-)_{in\ 1\ L} = 0.188$ mol

$$V(HC\ell) = \frac{n}{c} = \frac{0.188}{1.00} = 0.188 \text{ L} = 188 \text{ mL}$$

8. $pH = -\log_{10}[H^+]$
$5.50 = -\log_{10}[H^+]$
$[H^+] = 3.16 \times 10^{-6}$ mol L^{-1}
$n(H^+)_{in\ bore\ water} = cV = 3.16 \times 10^{-6} \times 15000 = 0.04743$ mol

$M(NaOH) = 39.998$ g mol^{-1}

$$n(OH^-)_{added} = n(NaOH) = \frac{m}{M} = \frac{10.0}{39.998} = 0.250 \text{ mol}$$

$H^+(aq) + OH^-(aq) \leftrightharpoons H_2O(\ell)$

$n(OH^-)_{in\ excess} = n(OH^-)_{added} - n(H^+)_{in\ bore\ water} = 0.250 - 0.04743 = 0.2025$ mol

$$[OH^-] = \frac{n}{V} = \frac{0.2025}{15000} = 1.351 \times 10^{-5} \text{ mol L}^{-1}$$

$k_W = [H^+]\,[OH^-] = 1.00 \times 10^{-14}$
$[H^+] \times 1.351 \times 10^{-5} = 1.00 \times 10^{-14}$
$[H^+] = 7.40 \times 10^{-10}$ mol L^{-1}
$pH = -\log_{10}[H^+] = -\log_{10}(7.40 \times 10^{-10}) = 9.13$

9. $pH = -\log_{10}[H^+]$
$7.80 = -\log_{10}[H^+]$
$[H^+] = 1.585 \times 10^{-8}$ mol L^{-1}
$n(H^+)_{\text{at pH 7.8}} = cV = 1.585 \times 10^{-8} \times 20.0 \times 10^6 = 0.317$ mol

$pH = -\log_{10}[H^+]$
$6.80 = -\log_{10}[H^+]$
$[H^+] = 1.585 \times 10^{-7}$ mol L^{-1}
$n(H^+)_{\text{at pH 7.8}} = cV = 1.585 \times 10^{-7} \times 20.0 \times 10^6 = 3.17$ mol

$n(HC\ell)_{\text{required}} = n(H^+)_{\text{required}} = n(H^+)_{\text{at pH 7.8}} - n(H^+)_{\text{at pH 7.8}} = 3.17 - 0.317 = 2.853$ mol

$$V(HC\ell) = \frac{n}{c} = \frac{2.853}{12.0} = 0.238 \text{ L} = 238 \text{ mL}$$

10. (a) $pH = -\log_{10}[H^+]$
$6.75 = -\log_{10}[H^+]$
$[H^+] = 1.78 \times 10^{-7}$ mol L^{-1}
$n(H^+)_{\text{at pH 6.75}} = cV = 1.78 \times 10^{-7} \times V$ mol

$pH = -\log_{10}[H^+]$
$5.10 = -\log_{10}[H^+]$
$[H^+] = 7.94 \times 10^{-6}$ mol L^{-1}
$n(H^+)_{\text{at pH 5.10}} = cV = 7.94 \times 10^{-6} \times V$ mol

$n(H^+)_{\text{total}} = n(H^+)_{\text{at pH 6.75}} + n(H^+)_{\text{at pH 5.10}}$
$= 1.78 \times 10^{-7} \times V + 7.94 \times 10^{-6} \times V = 8.118 \times 10^{-6} \times V$ mol

$$[H^+]_{\text{mixture}} = \frac{n}{V} = \frac{8.118 \times 10^{-6} \times V}{2V} = 4.059 \times 10^{-6} \text{ mol L}^{-1}$$

$pH = -\log_{10}[H^+] = -\log_{10}(4.059 \times 10^{-6}) = 5.39$

10 (b) $pH = -\log_{10}[H^+]$

$8.00 = -\log_{10}[H^+]$

$[H^+] = 1.00 \times 10^{-8}$ mol L^{-1}

$k_W = [H^+][OH^-] = 1.00 \times 10^{-14}$

$1.00 \times 10^{-8} \times [OH^-] = 1.00 \times 10^{-14}$

$[OH^-] = 1.00 \times 10^{-6}$ mol L^{-1}

$n(OH^-)_{at\ pH\ 8.00} = cV = 1.00 \times 10^{-6} \times V$ mol

From (a) $n(H^+)_{at\ pH\ 6.75} = cV = 1.78 \times 10^{-7} \times V$ mol

$H^+_{(aq)} + OH^-_{(aq)} \leftrightarrows H_2O_{(\ell)}$

So there will be an excess of OH^- ions

$n(OH^-)_{excess} = n(OH^-)_{at\ pH\ 8.00} - n(H^+)_{at\ pH\ 6.75}$

$= 1.00 \times 10^{-6} \times V - 1.78 \times 10^{-7} \times V = 8.22 \times 10^{-7} \times V$ mol

$$[OH^-]_{mixture} = \frac{n}{V} = \frac{8.22 \times 10^{-7} \times V}{2V} = 4.11 \times 10^{-7} \text{ mol L}^{-1}$$

$k_W = [H^+][OH^-] = 1.00 \times 10^{-14}$

$[H^+] \times 4.11 \times 10^{-7} = 1.00 \times 10^{-14}$

$[H^+] = 2.43 \times 10^{-8}$ mol L^{-1}

$pH = -\log_{10}[H^+] = -\log_{10}(2.43 \times 10^{-8}) = 7.61$